How long can I forget about my coffee?

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6 August 2021

1 Introduction

I am always forgetting about my coffee. I make it, wait for it to cool down, and by the time I remember to drink it, I have to heat it up again. In this project, investigated how well a double-walled vacuum thermos. Specifically, I answered the question: how long will a double walled thermos keep coffee in a drinkable temperature range?

I modeled the conduction through the vacuum walls of the thermos and convection through the imperfect vacuum between them. I ignored conduction through the lid for simplicity. Because heat transfer is impossible in a vacuum, I assumed that the thermos has an imperfect vacuum with a convection coefficient similar to air. I modeled the coffee thermos based on this 12oz example from Hydroflask. [2] The material of the thermos is 18/8 Pro-Grade Stainless Steel. By knowing both the density of the stainless steel and the weight of the thermos, I was able to estimate the thermos's wall thickness as well. The material properties of stainless steel were found from World Material. [3]

2 Geometry

The geometry modeled is a double-walled vacuum thermos. This is modeled as two nested cylinders. Each is a thin stainless steel wall that encapsulate an imperfect vacuum.

For simplicity, I am only including the walls of the cylinder and ignoring heat transfer through the top or bottom. Therefore the top and bottom of the cylinder are also not included in the model

I created a basic geometry in Fusion, a diagram of which is included in Figure 1.

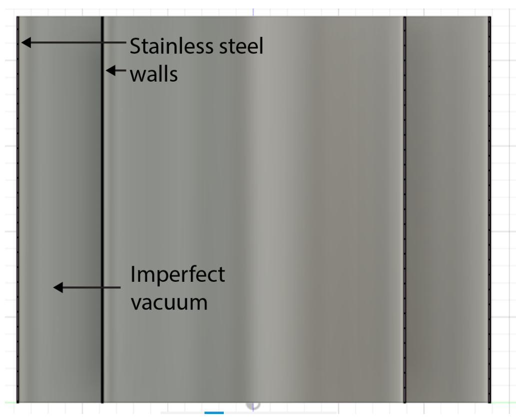

Figure 1: A cross-section of the model geometry

The geometrical parameters used for the model are outlined in Figure 2.

Parameter	Value
Inner diameter	8.89 cm
Outer diameter	14.0 cm
Height	11.4 cm
Weight	318 g
Wall thickness	0.0489 cm
Imperfect vacuum thickness	2.457 cm

Figure 2: Geometric parameters of model

3 Material Model

The first material of the model is 18/8 Pro-Grade Stainless Steel. This decision is based off the material used in a Hydroflask, a common commercial double-walled thermos. The relevant material value is thermal conductivity, with a value of $16.2\frac{W}{m*K}$. This value is obtained from World Material [3].

The second material of the model is the air in the imperfect vacuum between the stainless steel walls of the thermos. The relevant thermodynamic value for this material is the convection coefficient for which I chose the value $0.5\frac{W}{m^2*K}$. I chose this value based on the lower edge of the range of convection coefficients for air. I obtained information on the range of convection coefficients for air from Engineering Toolbox [6].

The third material of the model is the coffee in the thermos, simplified as water. Since I am modeling coffee at ”drinkable temperature,” or $50 - 60$°C [1], I chose a value of convection coefficient at the lower range of the convection coefficient of boiling water, $3.000\frac{W}{m^2*K}$. Again, I obtained information of the range of convection coefficients from Engineering Toolbox [6].

Parameter	Value
thermal conductivity of stainless steel	$16.2\frac{W}{m*K}$
specific heat capacity of stainless steel	$500\frac{J}{kg*K}$
density of stainless steel	$7.93\frac{g}{cm^3}$
convection coefficient of imperfect vacuum	$0.5\frac{W}{m^2*K}$
convection coefficient of drinkable coffee	$3.000\frac{W}{m^2*K}$
convection coefficient for air	$10.0\frac{W}{m^2*K}$

Figure 3: Table of relevant thermodynamic material parameters

Sink	Temp (° C)
Vacuum	20
Coffee	60

Figure 4: Table of sink temperatures for convection module

All these relevant material values can be found in Figure 3.

The modeling of convection in ABAQUS also requires that the temperature of the 'sink" be defined. In one case this will be the coffee, in another it is the imperfect vacuum. Both temperatures are defined in Figure 4.

4 Boundary Conditions

The boundary conditions of this model will be provided by the temperatures inside and outside the thermos. Because the air outside the thermos is at room temperature, the outer stainless steel wall or T_1 is 20°C. The coffee is assumed to be at the upper edge of the drinkable temperature range. Therefore the inner stainless steel wall or T_4 is 60°C. The drinkable range of coffee was obtained from Driftaway

Node	Temperature (°C)
T_1	20
T_4	60

Figure 5: Table of temperature boundary conditions

[1]. Temperature boundary conditions can be found in Figure 5.

5 Finite Element Mesh

For the mesh, I used quadrilateral, linear elements. To refine the mesh and perform a convergence study, I performed h-refinement, started at an approximate global size of 8, decreasing by half, and ending with an approximate global size of 0.1. Figure 6 shows an illustration of the coarsest mesh before analysis.

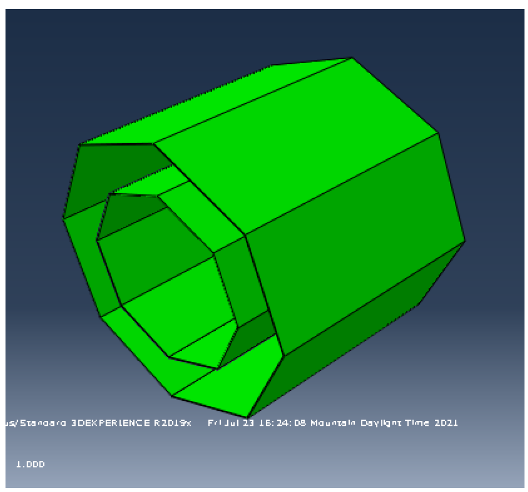

Figure 6: An illustration of the coarse mesh, global size 8

In my analysis I am interested in the length of time it takes for the temperature

Seed	Outer Temp (°C)	Inner Temp (°C)
16	15.87	60.00
8	15.87	60.00
4	15.57	60.00
2	15.40	60.00
1	15.35	60.00
0.5	15.33	60.00

Figure 7: H-refinement table

of the coffee, and thus the inner wall, to drop from 60 ° C to 50 ° C.

To obtain this information, I will need to know the temperature of the inner wall over time. I will need to perform a time-dependent study. However, for the convergence study, I will just be examining the first time step.

For my convergence study, I will be using the temperature of the outer wall at the first time step. This is because the inner wall temperature remains mostly constant until later.

To refine my mesh, I used h-refinement, starting with a global seed size of 8, and decreasing by half to 0.5. Figure 7 shows the temperature values corresponding to each seed and figure 8 shows a plot of the convergence.

Because of the thinness of the walls, even the coarser meshes provide good accuracy. For this reason, I chose a seed of 16 for my final model.

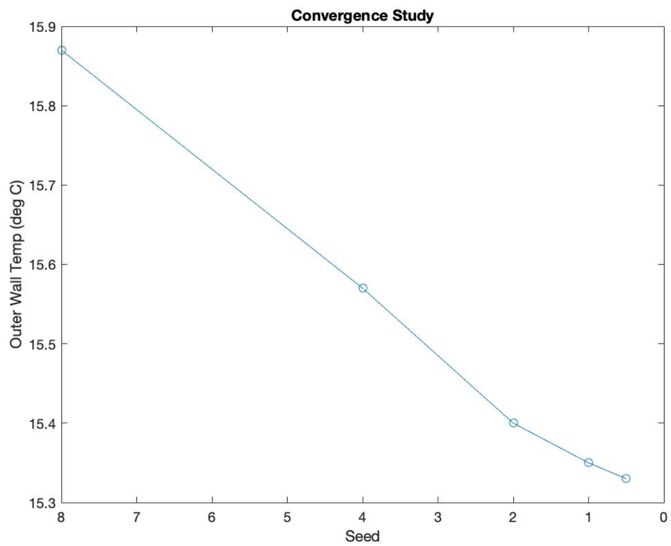

Figure 8: Convergence study plot

6 Verification

The model can be verified using a chain of transient conduction and convection equations as described below.

Transient 1D Heat Conduction equation: [4]

$$T_{cond1} = T_i + \frac{(T_0 - T_i)\delta}{\sqrt{\pi \alpha t}} e^{-\frac{\pi^2}{4\alpha t}} \tag{1}$$

δ, thickness $= 0.0489$ cm

$\alpha = \frac{k}{\rho * c_p} = 0.00409$

$T_0 = 60°\text{C}$

$T_i 1 = T_{conv}$

Transient 1D Heat Convection equation: [5]

$$T_{conv} = T_\infty + (T_i - T_\infty)e^{-\frac{hAt}{\rho Vc}} \tag{2}$$

A, area, $= 27.2cm^2$

h, convection coefficient, $= 0.5\frac{W}{m^2 * K}$

$rho = 7.93\frac{g}{cm^3}$

$V = 6.3892cm^3$

$c = 500\frac{J}{kg * K}$

$T_i = 20$°C

$T_\infty = T_{cond2}$

Second conduction equation:

$$T_{cond2} = T_i + \frac{(T_0 - T_i)\delta}{\sqrt{\pi\alpha t}}e^{-\frac{\pi^2}{4\alpha t}} \tag{3}$$

δ and α values remain the same as above.

$T_0 = 20$°C

$T_i = 20$°C

7 Validation

There is no data readily available to validate this model. In order to validate it using experimental data, an experiment could be run by collecting temperature data of the coffee using thermocouples. The thermocouples would be placed on the inner wall of the thermos. Water hotter than 60°C would be poured into the thermos and real-time data couple be collected once the lid of the thermos is replaced. This way, data could be collected over the range of $50 - 60$°C to validate the model's accuracy over that temperature range

8 Results

Figure 9 show the results of both the model and the verification equations described in the section above.

Time (hours)	Coffee Temp Theoretical	Coffee Temp Model (°C)
24	60	60
48	60	60
72	60	60

Figure 9: Temperature change over time

9 Discussion and Conclusion

The time-dependent data in Figure 9 shows that there is little to no change in the temperature of the coffee, even over a time scale of 72 hours. In practice, there would be temperature loss due to factors such as the removal of the lid and the actual amount of air in between the walls of the thermos.

But both the FEM model and the theoretical equations agree: I should be able to forget about my coffee for a very long time before it drops out of the drinkable range.

References

[1] Driftaway. *The Ideal Temperature to Drink Coffee*. URL: https://driftaway.coffee/temperature/. (accessed: 07.15.2021).

[2] Hydroflask. *12 oz insulated coffee mug*. URL: https://www.hydroflask.com/12-oz-coffee-mug?color=pacific. (accessed: 08.03.2021).

[3] World Material. *18/8 Stainless Steel Properties, Yield Strength, Composition, Density, Tensile Strength, Hardness*. URL: https://www.theworldmaterial.com/what-is-18-8-stainless-steel/. (accessed: 07.14.2021).

[4] MIT. "1D Heat Equation and Solutions". In: *3.044 Materials Processing* ().

[5] MIT. *Transient Heat Transfer (Convective Cooling or Heating)*. URL: http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node129.html. (accessed: 08.01.2021).

[6] Engineering Toolbox. *Convective Heat Transfer*. URL: https://www.engineeringtoolbox.com/convective-heat-transfer-d_430.html. (accessed: 07.14.2021).