How Many Engineers Does it Take to Change a Light Bulb?

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Abstract: In this model, we propose a closed-loop PID control design for inverted pendulum systems. With the example of a motorized cart being used to keep a ladder upright, we have created a model for creating a stable inverted pendulum system that minimizes external impulse on the system while maintaining stability. We then iteratively determined an optimal set of PID parameters to accomplish stability with low external impulse.

I. Design Proposal: The Magical Motorized Ladder

Say you want to change a lightbulb in the middle of the room, but it's in the middle of the room; you can't lean your ladder against the wall, so what do you do? The American Ladder Company has commissioned us to answer this very question. Our proposed design is an inverted pendulum that controls the ladder via a motorized cart. By applying variable forces, the motorized cart will keep the ladder upright while attached to the cart at only a pivot point.

Question
What system parameters will minimize the amount of total force while maintaining stability?

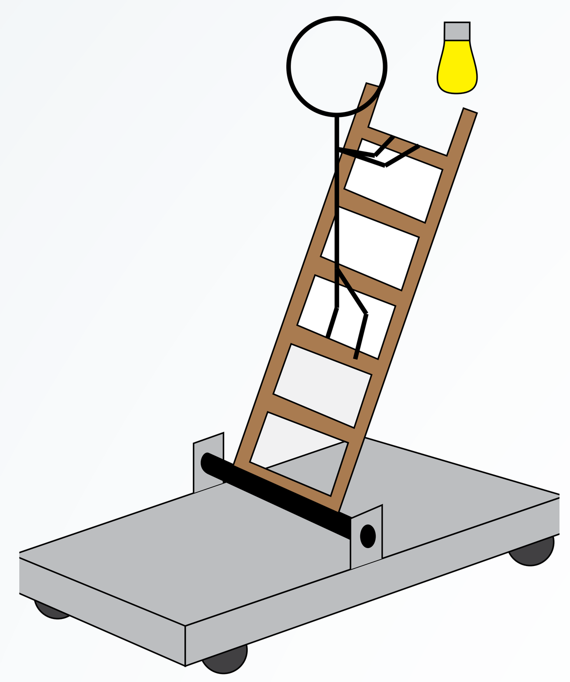

Figure 1: Simple diagram of person and ladder on cart

In order for the ladder to be stable, it needs to remain within **.5 radians** from upright in either direction, beginning with an offset of **.1 radians.**

II. Physical Inverted Pendulum Model

Fg = Force due to gravity

Ff = Force due to friction

N = Horizontal component of normal force at pivot point

P = Vertical component of normal force at pivot point

Fe = External force on system from motors

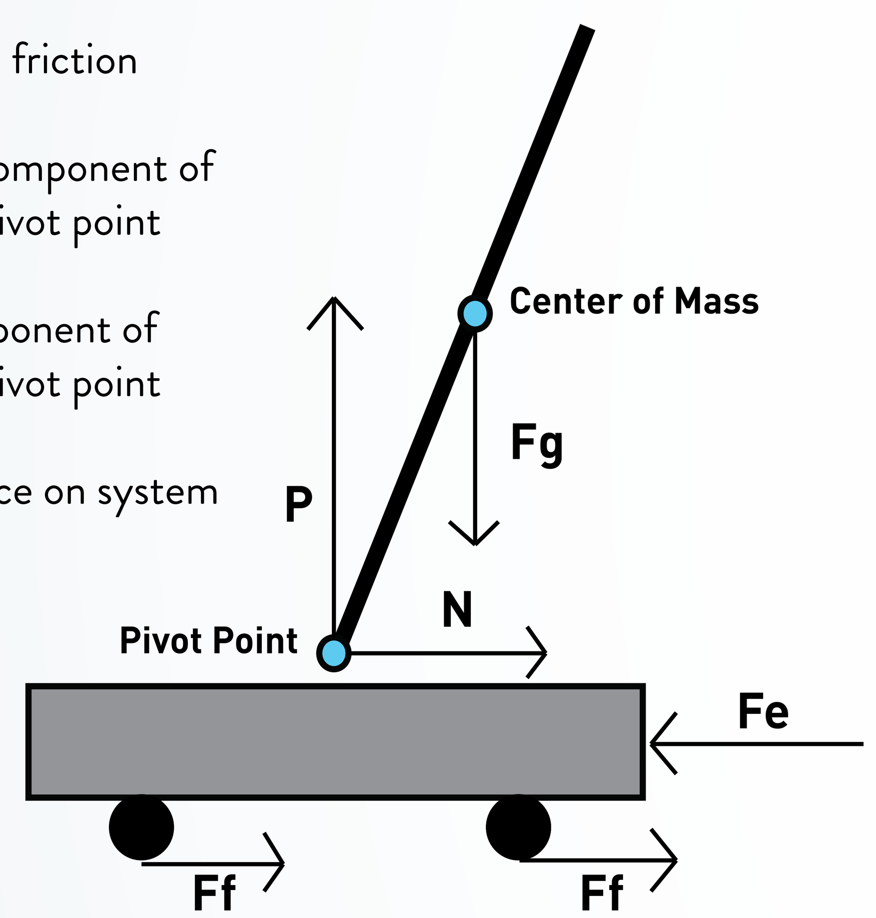

Figure 2: The free-body diagram of our system.

Equations of Motion

$$\ddot{x} = \frac{F - b\dot{x} + ml\dot{\theta}^2 sin\theta}{M + m - \frac{(mlcos\theta)^2}{I + ml^2}}$$

$$\ddot{\theta} = \frac{ml\ddot{x}cos\theta - mglsin\theta}{I + ml^2}$$

- b = coefficient of friction, $.5\ \frac{N}{m/s}$
- l = length to center of mass, $1.5\ m$
- M = mass of cart, $500\ kg$
- m = mass of pendulum, $148\ kg$
- I = moment of inertia, $112\ kg/m^2$

*Values for parameters taken from [3], [4]

III. Key Abstractions and Limitations of Model

Abstraction: Ladder and person as a single rigid mass
This assumption is critical for simplicity of the model, as it greatly simplifies calculations involving inertial moments and center of mass. In reality, the person will be constantly moving on the ladder, causing a constant change in the center of mass of the ladder/person system. Neglecting this change has a significant impact on the angular acceleration of the system.

Abstraction: No loss of energy due to internal functions of motorized system
In reality, controlling the speed of the motors is much less precise than a single force that can be accurately controlled; there are a number of internal losses due to friction, torque, etc.

IV. Varying External Force: Closed-Loop PID Control

In our system, the ladder is held upright by a computer-controlled, motorized cart. The acceleration is determined by the control of the motors - in our system, this is abstracted to a single external force. This force is varied at a constant timestep (10 ms) using a method known as PID control. **PID control** varies the external force on the cart using the equation found in Figure 3, which changes the force based on the error function, e(t), and three important **PID parameters:**

k_p - Proportional
k_i - Integral
k_d - Derivative

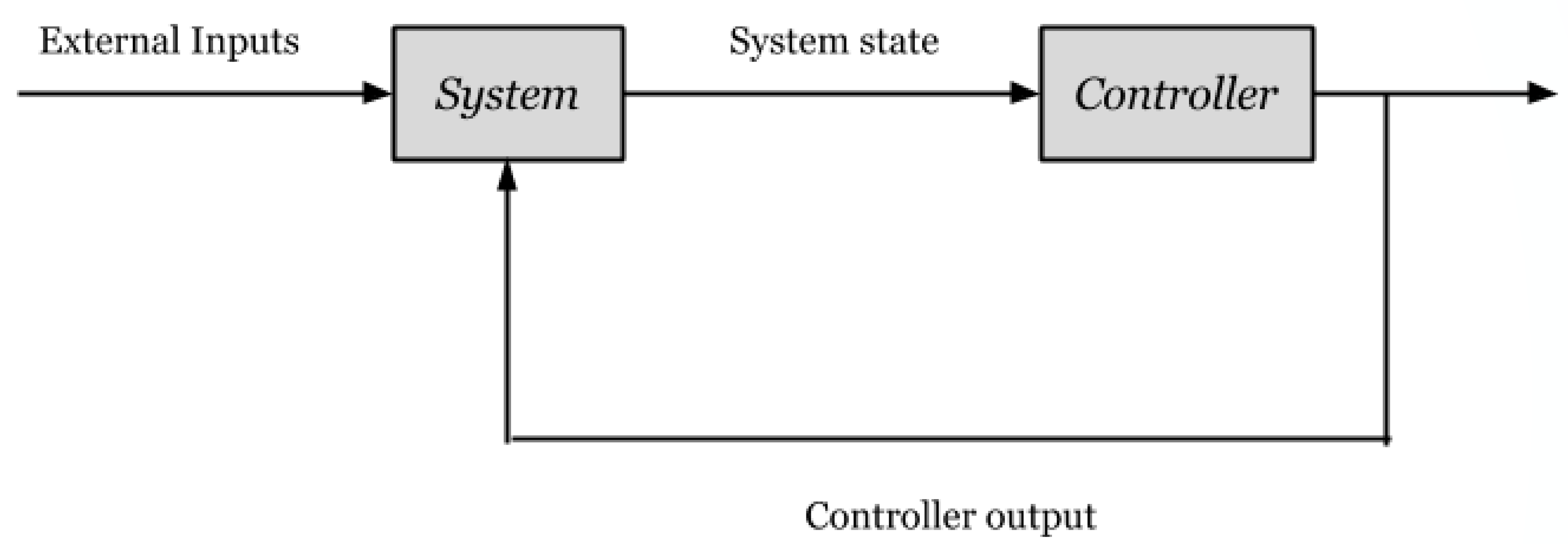

Figure 4: A diagram of the closed-loop PID control of the system

As these constants increase, total force on the system increases. There are many combinations of these three parameters that produce a fairly stable system, but increasing them will increase the total force that is applied. We want to minimize the values of these constants while remaining stable.

$$f_{out}(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{de(t)}{dt}$$

Figure 3: The governing equation for PID control.

V. Validating Closed-Loop Control

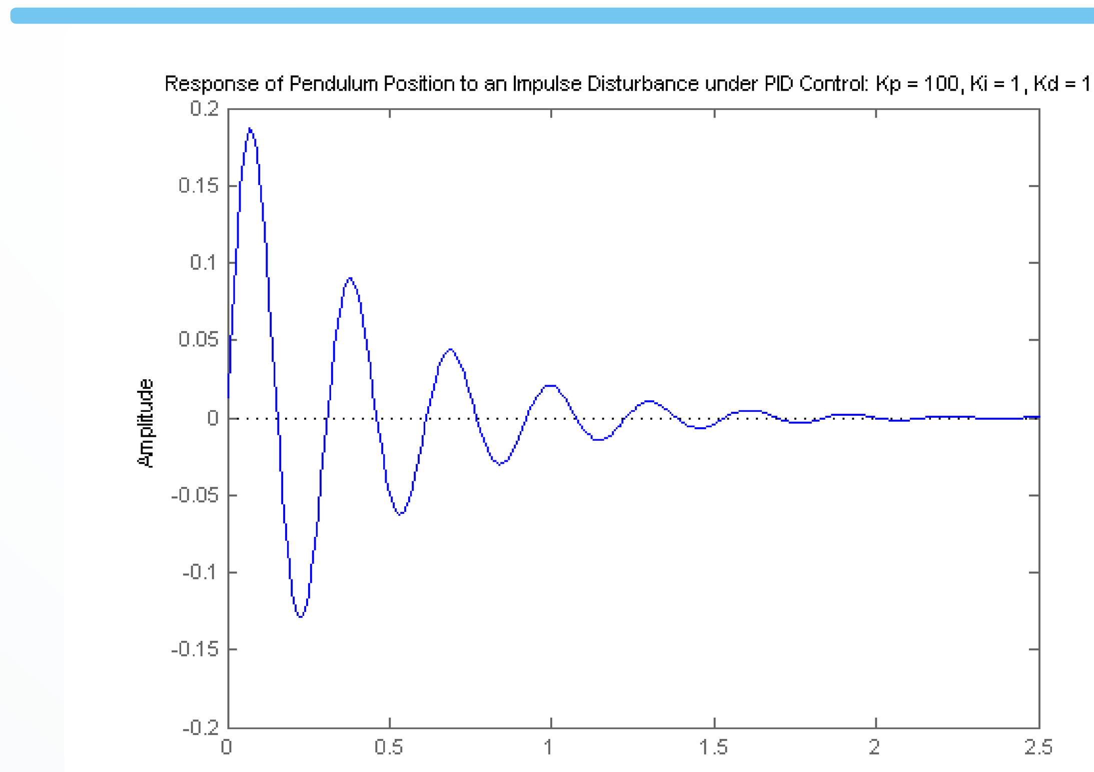

Figure 5: Response of simulated PID control, from source [1]

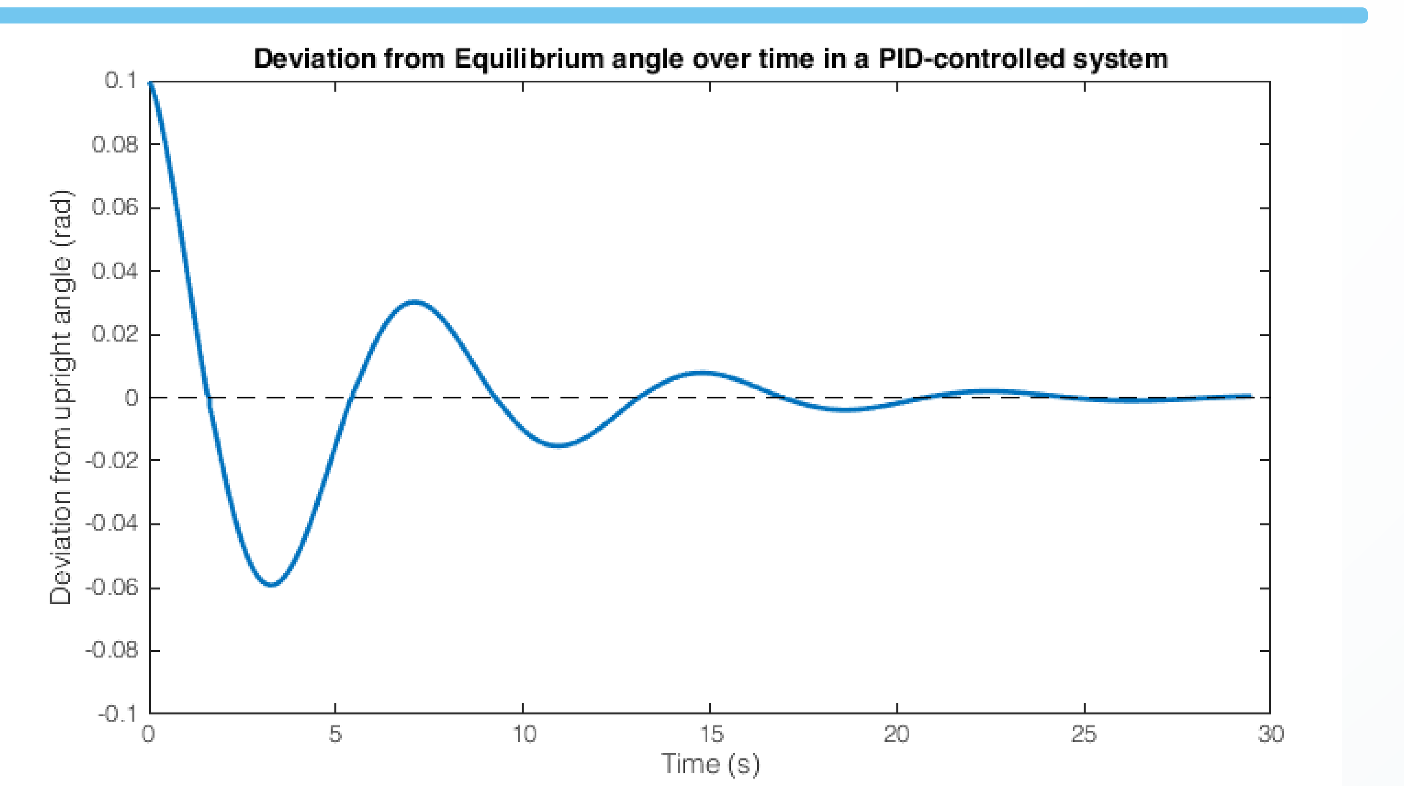

Figure 6: Response of simulated PID control from our model.

To validate the PID control of the system, we can compare our results (Fig. 6) to the results of a similar simulated system (Fig. 5). Qualitatively, these results are very similar, with some amount of dampened oscillation trending toward a stable system.

VI. Effects of Varying k_p and k_i on Stability of System

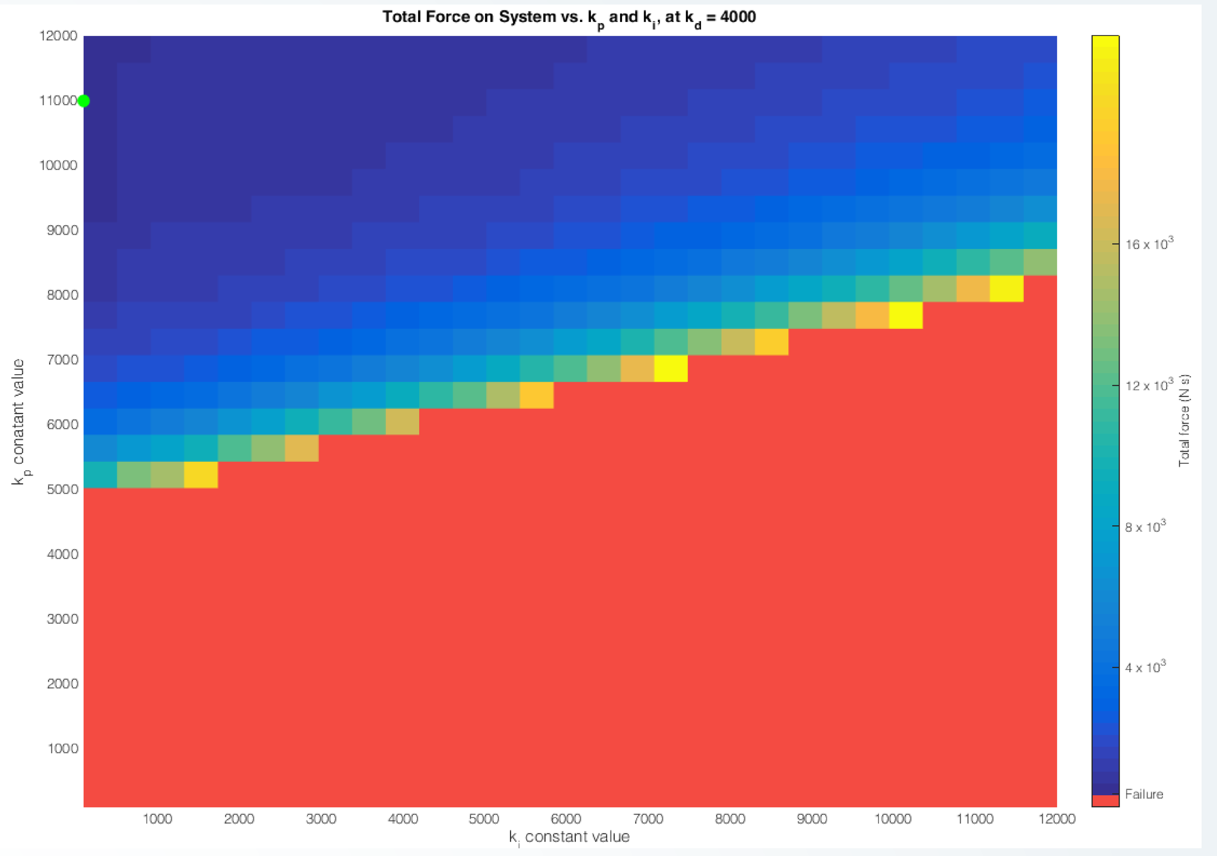

Figure 7. Plot of total force applied over t = 20 s at various values of k_p and k_i, at k_d = 4000.

In this diagram, red indicates points of failure; the green point marks the optimal combination of parameters.

VII. Optimal Parameters for System Stability

Based on the results of our simulation across a range of values of k_p and k_i with a constant k_d, we found that, when minimizing total force while maintaining stability, the optimal PID parameters are:

Optimal PID Parameters:
k_p = 11000
k_i = 100
k_d = 4000
f_{total} = 542.9 N s

Despite these "optimal" parameters, observing the system shows that it still doesn't behave optimally, particularly in the amount of horizontal displacement. Thus, other factors must be taken into account for true optimization of the system.

VIII. Next Steps

Incorporate shifts in center of mass
Because shifting center of mass in the person-ladder system was a critical limitation of the model, it would be important to incorporate this into future iterations of the model.

Develop further optimization for displacement
Horizontal is another important factor in the effectiveness of our system; thus, combining our current method of optimizing with a method that factors in horizontal displacement would be beneficial.

IX. Sources

- [1] J.R. White, "Introduction to the Design and Simulation of Controlled Systems". http://www.profjrwhite.com/system_dynamics/sdyn/s7/s7invp1/s7invp1.html
- [2] http://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum&section=SystemModeling
- [3] http://www.wernerco.com/us/support/ladder-safety-tips/how-to-choose-a-ladder
- [4] https://en.wikipedia.org/wiki/Human_body_weight